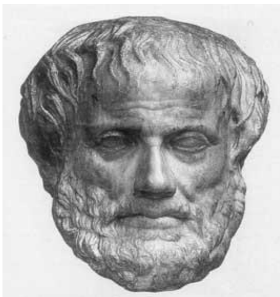
PHYS 3033 GENERAL RELATIVITY PART I

Chapter 1

The evolution of the concepts of space and time from Antiquity to Einstein

1.1 Space and time in Antiquity

a) The physical space in Antiquity



Aristotle (384-322)

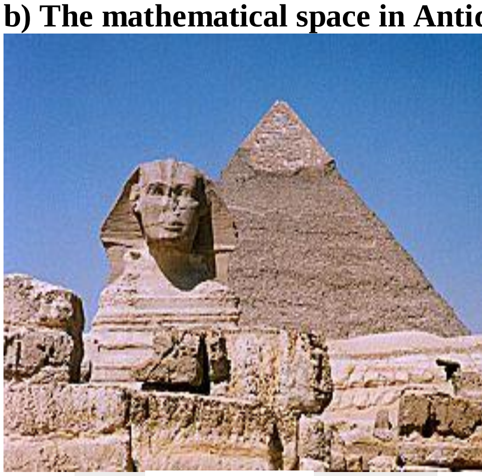
In Aristotle's (384-322 B.C.) days, ideas on physical space and time were vague and had yet to be sharpened into their modern forms[1]. Space was associated with the distribution of things directly observed. Things distributed in time, however, they were not directly observed, and generally intervals of time were not easily measured. How to define motion by combining intervals of space and time was not at all clear and motion was poorly distinguished from other forms of change.

In his book Physics (350 BC) (in Greek physis means nature, physicist means student of nature), Aristotle proposed a law of motion, which may be expressed by the relation:

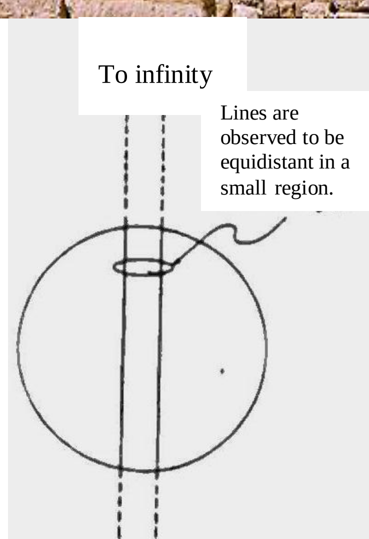
Applied force= resistance x speed

"A body will move through a given medium in a given time and through the same distance in a thinner medium in a shorter time".

b) The mathematical space in Antiquity



In the ancient Delta civilizations, geometry was the art of land measurement, indispensable in the construction of such mammoth works as the Great Pyramid of Giza and Stonehenge. The Babylonians of 2000 BC and the Chinese of 300 BC used the rule that the circumference of a circle is three times its diameter and this value for π is found in Hebraic scripture. The Greeks, in their thorough fashion, developed geometry into a science that climaxed in the axiomatic and definitive treatment presented by Euclid (325-265) at the Museum in Alexandria in the third century BC.



In his textbook, *The Elements*, he recorded in a systematic manner the advances in geometry made since the time of Thales. Euclid used five basic axioms and from them, with accompanying definitions, deduced all that was known in geometry in 465 theorems[2].

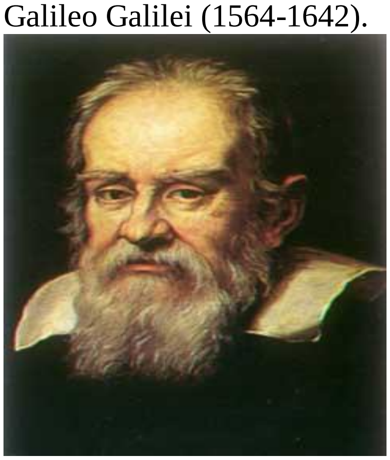
The axiom of most interest to us, which remained controversial until the nineteenth century, is the Euclidian parallel (fifth) postulate.
The definition of parallel states that two straight lines, drawn in the same plane, are parallel if they do not intersect.
The fifth postulate asserts that through any point there is one and only one parallel to a given straight line.

For more than 2000 years most persons accepted the fifth postulate as obvious. A few, however, including Euclid, confessed uneasiness because the postulate cannot be verified by direct appeal to experience. Many geometricians sought for a more basic axiom from which the parallel postulate could be derived. But all attempts failed.

1.2 Space and time in the Modern Age

With the decline of the Greek culture, scientific modeling came to a halt. Greek learning was preserved by the Arabs, who added further observations to the growing volume of scientific data. Some Arab scholars were dissatisfied with Aristotelian physics, but no new theories arose in the Middle Age. Thomas Aquinas (1226-1274) and other medieval theologians elevated the Ptolemaic cosmology and Aristotelian physics into a cornerstone of Christian doctrine.

Human curiosity cannot be suppressed, however. The rediscovery of the Greek scientific thoughts began a transformation in Europe that led eventually to the Renaissance. During this period the increased level of literacy and education, the rediscovery of ancient scholarly works and the development of printing raised the intellectual standards and altered the political climate. The name most popularly associated with the championing of this new world view is that of the Italian astronomer Galileo Galilei (1564-1642).



Galileo Galilei (1564- 1642)

Galileo has devoted much of his career to the physics and mechanics. In particular, he was intrigued by the motion of falling bodies. Aristotle held that the rate of fall depends upon the composition of the falling body and of the medium through which the body fell. Galileo recognized that this idea could be put to the test. He carried out his own experiments and made measurements in support of his conclusion that all objects fall at the same rate, contrary to the Aristotelian claim. The limitations of the technology of his time forced him to appeal for many of his arguments to thought experiments, that is, mental experiments that could, in principle, be performed if the technology were available.

Thus he concluded that all objects must fall at the same rate in the vacuum.

This important observation is called now the equivalence principle and it became the basis of the Newton's theory of gravity and also for Einstein's theory of relativity.

A key rule of mechanics is the law of inertia. The essential break from Aristotelian mechanics to modern mechanics is to recognize that force is responsible not for motion, but for changes in motion. From this realization, the relativity of uniform motion follows. *Galileo understood the experimental fact that if everything moves together uniformly, such as the furniture and lamps in the interior of a moving ship, then it will seem no different from when the ship is at dock.*

Galileo summarized his conclusions in 1632 in a new book entitled "Dialogo de due Sisteme Mundi" (Dialogues Concerning the Two Chief World Systems). The book cause sensation throughout educated Europe and paved the way for the new paradigm of the Universe. It also set the stage for Galileo's later troubles with the Church. One of his political missteps was to place the defense of the Aristotelian cosmology into the mouth of Simplicio, an obvious fool[3].

Galileo never completely worked out the laws of motion that would replace those of Aristotle. That task fell to Isaac Newton.

Isaac Newton (1642-1726)

Isaac Newton[4] (1642-1726), one of the most illustrious of all scientists, gathered together the thoughts of many thinkers since the Middle Age. The genius of Newton, meditating for many years on the natural philosophy of space, time and motion, transformed all previous graphical descriptions into mathematical prescriptions. In his "Principia Mathematica Philosophiae Naturalis" (Mathematical Principle of Natural Philosophy), Newton said of space:

"*Absolute space, in its own nature, without relation to anything external, remains always similar and immovable*."

Of time, he said:

"*Absolute, true and mathematical time, of itself, and from its own nature, flows equally without relation to anything external*."

These thoughts of an absolute space, of a space existing in its own right without need of material support, at that time seems contrary to the common sense. But Newton believed that he had proof of absolute space.

Newton's three laws of motion state:

1. *A body continues in a state of rest, or of constant motion in a straight line, unless compelled to change that state by an applied force.*
2. *The rate momentum changes in time equals the applied force and is in the direction of the force.*
3. *To every force there exists at the same place and time an equal and opposite force.*

By using these laws, Newton showed that the gravitational attraction between any two bodies varies as the inverse square of their separating distance. He also showed *that a spherical body exerts a gravitational attraction as if all its mass was concentrated at the center of the body and that the natural orbits of the planets are ellipses. The orbits of all bodies freely moving in the Sun's gravitational field are ellipses, parabolas or hyperbolas*.

Newton and *Gottfried Leibniz* (1646-1716), a German philosopher, independently developed the mathematical principles of calculus.

Absolute space and time were the overarching concept of the Newtonian Universe. This was implicit in the Newtonian equation of motion. However, many scientists regarded them as wrong.

Leibniz regarded Newton's idea as outrageous and asserted:" *There is no space where there is no matter*."

The idea of absolute space was vigorously attacked by the Irish philosopher *George Berkeley* (1685-1753) in a work entitled *Motion* (1721). He stated that a single body, in an otherwise empty universe has no measurable motion of any kind. Ideas similar to Berkeley expressed *Ernst Mach* (1838-1916), an Austrian physicist whose work deeply influenced Einstein. He explained the inertial force as determined by and proportional to the total amount of matter in the universe. Hence the universe of stars is responsible for the inertial forces of non-uniform motion.

1.3 Birth of non-Euclidian geometry

While in the 19th century the concepts of absolute space and time dominated the physics, a revolution started in the understanding of the mathematical nature of space.

The famous German philosopher Immanuel Kant (1724-1804) shared the prevailing belief that Euclidian geometry is transparently true and no alternative system of geometry is conceivable by the human mind. In his basic work Critique of Pure Reason he attempted to place Euclidean geometry on a firm basis by arguing that its axioms are "a priori" (prior to experience) and "an inevitable necessity of thought." Kant believed that what is unimaginable is automatically impossible.

But in mathematics and physics what is possible today was unimaginable yesterday.

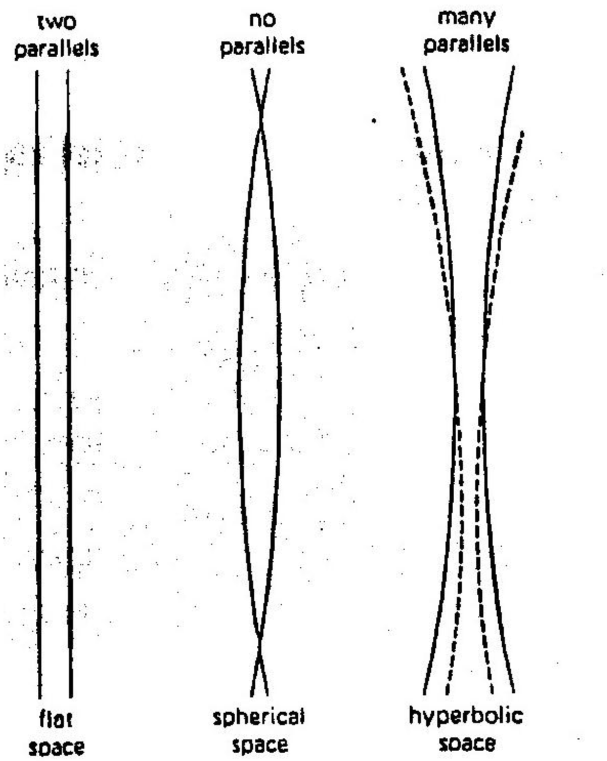
We know that the parallel postulate is a fundamental statement and cannot be reduced to a more basic axiom. It singles out Euclidian space from other possible spaces. In Euclidian space the circumference of a circle is π times its diameter and the sum of the interior angles of a triangle is equal to two right angles. In other spaces these relations are not necessarily true.

The first examples of non-Euclidian spaces were found by Janos Bolyai (1802-1860) and Nikolai Lobachevski (1793-1856). They form the hyperbolic geometry. Another class of geometries-the spherical geometry-was discovered by Georg Bernhard Riemann (1826-1866). All these works were based on the remarkable previous researches of Karl Friederich Gauss[5] *(1777-1855).*

Karl Friederich Gauss (1777-1857)

Nikolai Ivanovich Lobachevsky (1793-1856)

Janos Bolyai (1802-1860)



Parallel lines in uniform spaces.
Through a given point: (a) in flat space there is only one parallel to a straight line; (b) in spherical space there are no parallels to a weight line; and (c) in hyperbolic space there are many parallels to a straight line.

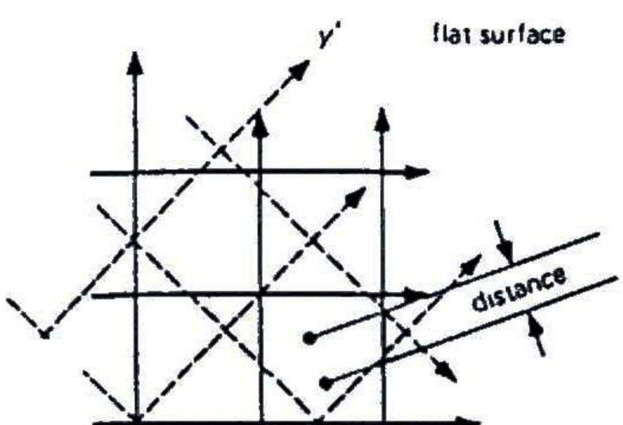
The non-Euclidean spaces are defined by the following postulates:

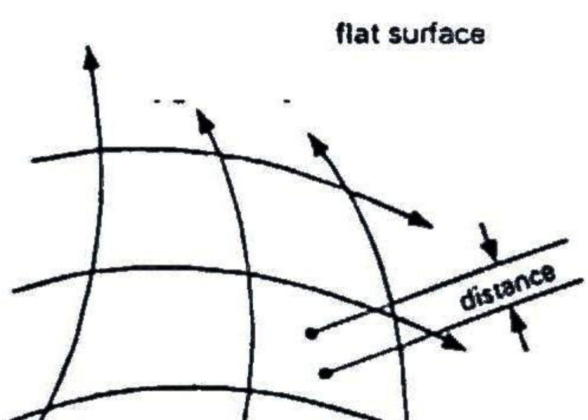
a) in the Euclidian space there is one parallel to a straight line through a given point

b) in the spherical space there is no parallel to a straight line through a given point.

c) in the hyperbolic space there are many parallels to a straight line through a given point

To explain these geometries, we start with a flat surface and lay out a network of imaginary lines that form a coordinate system. If the coordinates are perpendicular to each other, and labeled x and y, the distance Δs between any two points is given by the Pythagorean rule





(a) (b)

Coordinate systems in flat surfaces. When coordinate lines are perpendicular to each other, as are x and y or x' and y' shown in a), the distance between any two points is given by the ordinary Pythagorean rule. When coordinates are curvilinear, as shown in b), and not necessarily perpendicular, the distance between two adjacent points is given by a more general rule involving metric coefficients that vary from point to point and depend on the arbitrary coordinates chosen.

$$(\text{space interval})^2 = (x\,\text{interval})^2 + (y\,\text{interval})^2 \Leftrightarrow \Delta s^2 = (\Delta x)^2 + (\Delta y)^2$$

We may use any coordinate system consisting of a network of arbitrarily curved lines and the Pythagorean rule becomes

$$(\text{space interval})^2 = F(x\,\text{interval})^2 + 2G(x\,\text{interval} \times \text{y interval}) + H(y\,\text{interval})^2,$$
$$(\Delta \text{s})^2 = F(x,y)(\Delta x)^2 + 2G(x,y)(\Delta x \times \Delta \text{y}) + H(x,y)(\Delta y)^2$$

where F, G, H are functions of x and y. An equation of this type, which gives the distance between two adjacent points, is known as a metric equation and the functions F, G, H are the metric coefficients.

The basic mathematical concept describing the deviation from the Euclidian geometry is the curvature K. It is defined as

$$\text{sum of interior angles of a triangle} - \pi = K \times \text{area of triangle}$$

These rules enable us to determine the curvature anywhere in any space of two or more dimensions. We have

Spherical space (closed): K is positive

Euclidian space (open): K =0

Hyperbolic space (open): K is negative

B. Riemann (1826-1866)

Riemann, in his famous inaugural doctoral lecture, "*On the hypothesis forming the foundation of geometry*", generalized the approach to geometry of Bolyai, Lobachevski and Gauss. He defined the distance between adjacent points by the Pythagorean rule, with arbitrary metric coefficients, which vary from place to place. He developed the differential equations for the variation of the metric coefficients. One of these equations gives us the Riemann curvature, which is more general than the curvature K. In a two-dimensional curved space the curvature has one value. In three-dimensional space it has 6 components and in a four dimensional space 24.

Einstein paid to Riemann the tribute: "*Only the genius of Riemann, solitary and uncomprehended, had already won its way by the middle of the last century to a new conception of space, in which space was deprived of its rigidity and in which its power to take part in physical events was recognized as possible*."

1. 4 Space and time in relativity

a) Unification of space and time in special relativity

In the 1860's the British physicist James Clerk Maxwell (1831-1879) developed a theory of electricity and magnetism, which showed that these two forces were actually manifestations of one "electromagnetic" force. A consequence of Maxwell's equations was that fluctuating, time varying electromagnetic fields traveled through space at the speed of light. It soon became clear that this electromagnetic radiation was light itself. Waves in matter, such as elastic waves or sound waves, require a medium in which to propagate.

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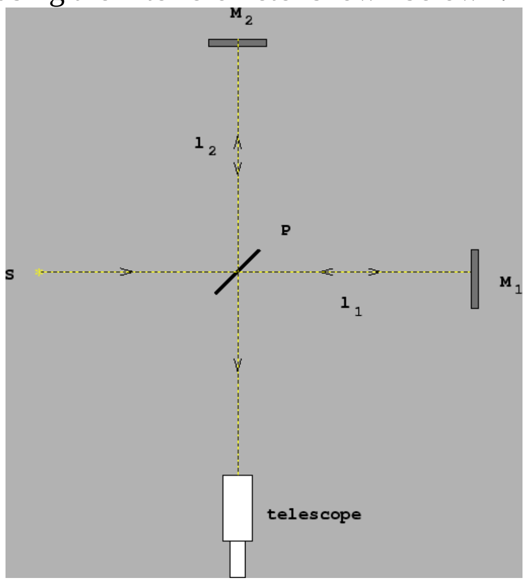
James Clerk Maxwell (1831-1879)

One of the most important physicists of all times, Maxwell unified electricity and magnetism in a single theory, showing that they are different manifestations of the electromagnetic field. The four differential equations he proposed for the description of the electromagnetic field are among the greatest achievements in the history of science. From these equations Maxwell deduced that light is an electromagnetic phenomenon in its nature. He also made important contributions in astronomy, statistical mechanics, thermodynamics, heat theory and mathematics.

In the 19th century scientists concluded that light too traveled through a medium, called the luminiferous ether or just the ether. The light waves always move with the speed of light relative to the ether.

In 1887 Albert Michelson and Edward Morley found that the speed of light is the same in all directions on the Earth surface. This was an unexpected result. The Earth moves at orbital speed $v = 30 km/s$ *around the Sun and Michelson and Morley expected to find that the measured speed of light would be* $c + v$ *and* $c - v$ *in opposite directions parallel to the Earth's orbital motion. Instead, they found that the speed of light is the same in both directions*.

In principle, it should be possible to detect the motion of the Earth through the ether, if the latter exists and is not dragged along with the Earth. The most sensitive search for this relative motion was done at the end of the nineteenth century by Michelson and Morley, using the interferometer shown below[6].



A simplified version of the Michelson interferometer

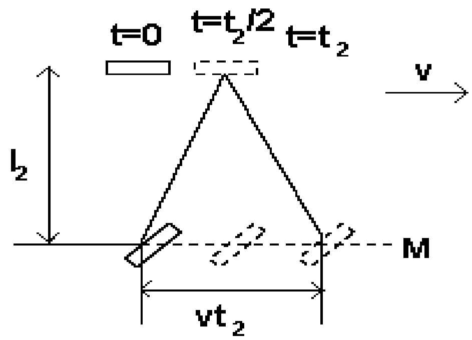
The Michelson interferometer is fixed on the Earth. If we imagine the ether to be fixed with respect to the Sun, then the Earth (and the interferometer) moves through the ether at a speed of 30 km/s, in different directions in different seasons.

The beam of light from the laboratory source S is split by the partially silvered mirror P, inclined 45 degrees to the beam's direction, in two coherent beams, beam 1 being transmitted through P and beam 2 being reflected off P. Then the returning beam 1 is partially reflected and the returning beam 2 is partially transmitted by P, back to the telescope, where they interfere. The interference is constructive or destructive depending on the phase difference of the beams. If M_1 and M_2 are very nearly at right angles, a fringe system will form in the telescope, consisting of nearly parallel lines.

Let us compute the phase difference between the beams 1 and 2. This difference can arise from two causes: the difference path lengths traveled, l_1 and l_2, and the different speeds of travel with respect to the instrument because of the ether wind v. The time for beam 1 to travel from P to M_1 is

$$t_1 = \frac{l_1}{c-v} + \frac{l_1}{c+v} = \frac{2l_1}{c} \frac{1}{1 - \mathrm{v}^2 / c^2} .$$

To calculate the time for the beam 1 we assumed that light, whose speed in ether is c, has an "upstream" speed of $c - \mathrm{v}$ with respect to the apparatus and a "downstream" speed of $c + \mathrm{v}$. The path of beam 2, traveling from P to M_2 and back, is a cross-stream path through the ether.



The transit time is given by

$$2\left[l_2^2+\left(\frac{\mathrm{v}t_2}{2}\right)^2\right]=ct_2,$$

or

$$t_2=\frac{2l_2}{c}\frac{1}{\sqrt{1-\mathrm{v}^2/c^2}}.$$

The difference in transit times is

$$\Delta t=t_2-t_1=\frac{2}{c}\left(\frac{l_2}{\sqrt{1-\beta^2}}-\frac{l_1}{1-\beta^2}\right),$$

where we denoted the dimensionless parameter v/c by β.

Suppose that the instrument is rotated through 90 degrees, thereby making l_1 the cross-stream length and l_2 the downstream length. If the corresponding times are now designated by primes, the same analysis as above gives the transit-time difference as

$$\Delta t'=t_2'-t_1'=\frac{2}{c}\left(\frac{l_2}{1-\beta^2}-\frac{l_1}{\sqrt{1-\beta^2}}\right).$$

Hence the rotation changes the differences by

$$\Delta t' - \Delta t = \frac{2}{c}\left(\frac{l_2 + l_1}{1-\beta^2} - \frac{l_2 + l_1}{\sqrt{1-\beta^2}}\right) \approx \frac{2}{c}(l_2 + l_1)\left(1 + \beta^2 - 1 - \frac{1}{2}\beta^2\right) = \frac{l_1 + l_2}{c}\beta^2 .$$

To obtain the above result we have used the binomial expansions

$$\frac{1}{1-\beta^2} = 1 + \beta^2 + \beta^4 + \beta^6 + ..., \frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{1}{2}\beta^2 + \frac{3}{8}\beta^4 + \frac{5}{16}\beta^6 + ...$$

and dropped terms higher than the second order.

If the optical path difference between the beams changes by one wavelength, for example, there will be a shift of one fringe across the crosshairs of the viewing telescope. Let ΔN represent the number of fringes moving past the crosshairs as the pattern shifts. Then, if light of wavelength λ is used, so that the period of one vibration is $T = 1/\nu = \lambda / c$,

$$\Delta N = \frac{\Delta t' - \Delta t}{T} \approx \frac{l_1 + l_2}{cT}\beta^2 = \frac{(l_1 + l_2)\beta^2}{\lambda}.$$

Michelson and Morley used an optical path length $l_1 + l_2$ of 22 m, with the two arms (nearly) equal, $l_1 = l_1 = 11$ m. For $\lambda = 5.5 \times 10^{-7}$ m and $\beta = 10^{-4}$,

$$\Delta N = 0.4,$$

or a shift of four-tenths a fringe.

Michelson and Morley mounted the interferometer on a massive stone slab for stability and floated the apparatus in mercury, so that it could be rotated smoothly. In order to make the light path as long as possible, mirrors were arranged on the slab to reflect the beams back and forth through eight round trips. The fringes were observed under a continuous rotation of the apparatus, as a shift as small as 0.01 of a fringe definitely could have been detected. Observations were made day and night and during all seasons of the year. The experimental conclusion was that *there was no fringe shift at all*. Therefore the experiment showed no indication of any movement of the Earth through the ether.

Interference pattern from the Michelson-Morley interferometer. The pattern is independent on the motion of the Earth in the space.

A possible explanation of the null result of the Michelson-Morley experiment was suggested by Lorentz and Fitzgerald, who advanced the hypothesis that all bodies are contracted in the direction of motion relative to the stationary ether by a factor $\sqrt{1-\mathrm{v}^2/c^2}$. But other experiments have shown that this hypothesis is not enough to give a full explanation of the null result.

Therefore from the result of the Michelson-Morley experiment we can firmly conclude that

the velocity of light is independent of the velocity of its source, of the observer, and of the nature of light itself.

The constancy of c for all observers, even for observers moving in opposite directions at high speed is greatly astonishing. Suppose a space-ship is moving at half the speed of light. Relative to that ship light has the same speed c that it has relative to us.

Albert Einstein (1879-1955)

Albert Einstein[7] (1879-1954), a young physicist from Berna, Switzerland, considered the implications of this experimental result, in their full generality. He had the audacity and courage to abandon the Galilean relativity, and with it Newtonian mechanics.

He proposed two relativity postulates:

1. *The laws of Nature are the same in all inertial frames of reference.*

2. *The speed of light in vacuum is the same in all inertial frames of reference.*

Despite their simplicity, the two relativity postulates contain remarkable, even incredible, consequences, like time dilation or length contraction. Time intervals, space separation and simultaneity are not absolute. Space and time are relative to the observer and a measurement of time depends on space and vice-versa. In the theory of relativity we cannot think of “space” and “time” as separate entities.

The idea of space-time was developed by Hermann Minkowski (1864-1909*) in 1908 as a way to unify the mixing of space and time. Intervals of space and time are no longer invariant for all observers. Together they form an invariant space-time interval*

$$(\text{space - time interval})^2 = (\text{time interval})^2 - (\text{space interval})^2$$

$$(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 - (\Delta y)^2 - (\Delta y)^2$$

Notice that if time is measured in seconds (or years), then the distance is measured in light seconds (or light years).

Space and time are fused together to form a unified four-dimensional space-time continuum and, as Minkowski said, “*Henceforth, space by itself and time by itself are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality*.”

This interval must be compared with the usual distance between two points

$$(\text{space interval})^2 = (x\,\text{interval})^2 + (y\,\text{interval})^2$$

The space-time interval is the same for all observers, independent of their velocity. The minus sign (instead of a plus sign), accounts for the results of special relativity.

We are accustomed to the idea that the shortest distance between two points is a straight line. This is true in ordinary space but not in space-time. Let us consider the light cones that are the paths on which light rays travel. In 1 second of time light travels 1 light second in space. For a light ray this means

$$\text{interval of time} = \text{interval of space}$$

and therefore the space-time interval between two events on a light cone is zero.

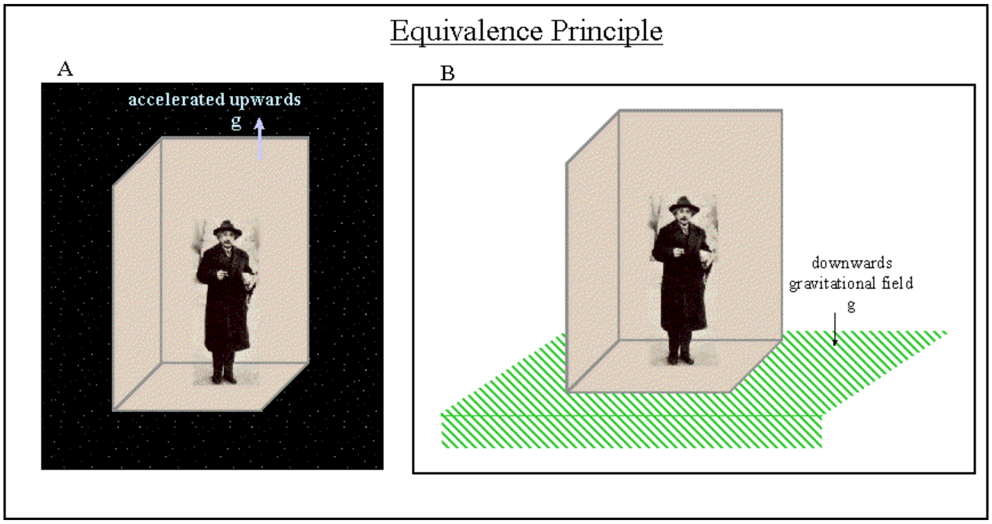
Consider a star at a distance of 1000 light years. Light emitted by this star comes to us in 1000 years. No one can disagree that the light has traversed immense intervals of space and time. But the space-time interval between the eye and the star is zero! These may seem bizarre but it explains why the speed of light is invariant and forms an upper limit to all material motion.

b) Geometry and gravitation

After 1909 Einstein started to think about the generalization of the special relativity to non-inertial frames and the gravitational force. Although he arrived quickly at the physical foundations of what became the general theory of relativity, the mathematical representation of the ideas was far from being obvious. Finally, around the time of the First World War, his friend Marcel Grossman introduced him to Riemannian geometry. Einstein found his answers there and he published the equations of general relativity in 1916 [8] .

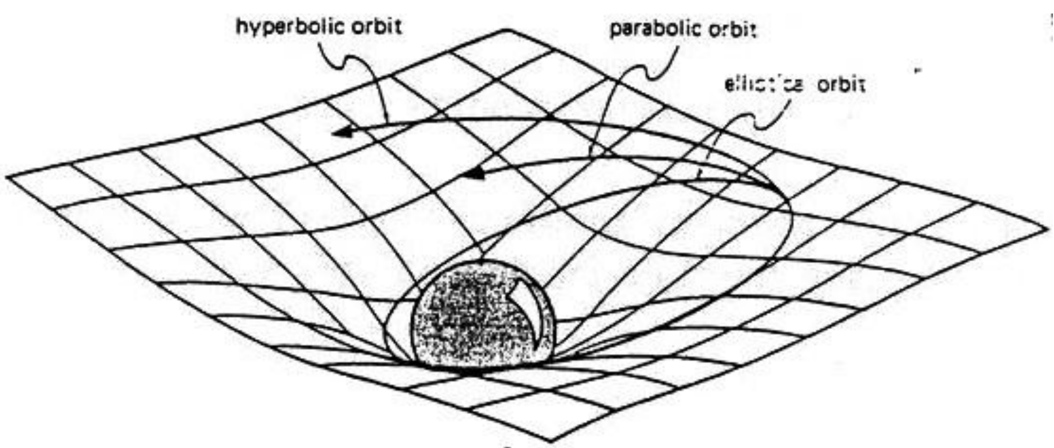
The general theory of relativity- a theory of gravity has two basic principles:

a) The principle of equivalence states that inertial and free-falling systems are entirely equivalent. The acceleration of a free-falling laboratory cancels completely the effect of gravity, not only dynamically, but also in all conceivable physical experiments.



The force on Albert, when the elevator is accelerating him, is the same, and has the same effect, as his being accelerated by an equivalent gravitational field.

If the principle of equivalence is the first stepping-stone to general relativity, the second is the realization that geometry and gravity have much in common.



. A horizontal, stretched rubber sheet is depressed by a heavy spherical body. The curvature of the sheet mimics the effect of gravity, and a ball bearing follows an orbit that is either elliptical, parabolic, or hyperbolic.

Let us consider a large rubber sheet initially flat. Its curvature is everywhere zero and it is like the flat space-time that exists far from a star where gravity is practically zero.

In the center of the sheet we now place a heavy ball that produces a large depression.

Far from the central body the surface is still flat and a ball follows a path that is almost straight.

Close to the central body the curvature of the surface is large and a ball bearing in motion accelerates in a way analogous to the acceleration of a body in the vicinity of a star.

By altering the initial speed of the ball we can make it describe elliptical-like, parabolic-like and hyperbolic-like orbits about the central body. In this way the curvature of the surface mimics the properties of gravity.

In Einstein theory of General Relativity the Newtonian Universe with its Euclidian geometry and gravitational forces is replaced with a relativistic Universe of space-time of varying curvature. The curved orbits of free-falling bodies in Newtonian Universe became the straights orbits in the curved space-time of the Einstein Universe.

b) The Einstein equations of General Relativity states that the curvature of the space-time is influenced by matter. The deformation of the space-time is related to the stress induced by matter. The equations states

$$\text{curvature of space - time} = \text{constant x matter}$$

We interpret the Einstein equations to mean that curvature is equivalent to gravity. The matter of the right side includes all forms of energy (including pressure) that have mass.

A paraphrase of Einstein’s theory (to remember if all else is forgotten)

“Matter tells space-time how to curve…

and space-time tells matter how to move.”

$$\boxed{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = -\frac{8\pi G}{c^4} T_{\mu\nu}}$$

These are Einstein's gravitational field equations, with $R_{\mu\nu}$ the Ricci tensor, $g_{\mu\nu}$ the metric tensor and R the scalar curvature. $T_{\mu\nu}$ is the energy-momentum tensor of the matter.

Notes

[1] For a discussion of Aristotle's contribution to philosophy and science see A. Edel, Aristotle and his philosophy, London, Croom Helm, 1982.

[2] The Greek science and mathematics is presented in detail in T. E. Rihll, Greek science, Oxford, Oxford University Press, 1999.

[3] The life of Galileo is described in P. Hightower, Galileo: astronomer and physicist, Springfield, Enslow Publishers, 1997; for the trial of Galileo see M. A. Finnocchiaro, The Galileo affair: a documentary history, Berkeley, University of California, 1989.

[4] A good presentation of Newton's life and scientific activity is given in P. Strathern, Newton and gravity, New York, Doubleday, 1998.

[5] For Gauss's life and his role in the creation of non-Euclidian geometries see W. K. Buhler, Gauss: a biographical study, Berlin, Springer 1981.

[6] The presentation of the Michelson-Morley experiment closely follows the discussion in R. Resnick and D. Halliday, Basic concepts in relativity and early quantum theory, McMillan Publishing Company, New York, 1992.

[7] For a recent biography of Einstein see D. Brian, Einstein: a life, New York, J. Wiley, 1996. A fascinating description of Einstein's life in the years before 1905 is given in M. Zackheim, Einstein's daughter: the search for Lieserl, New York, Riverhead Books, 1999.

[8] An excellent presentation of the historical development of the special and general relativity is given in J. Mehra, The golden age of the theoretical physics, Singapore, World Scientific, 2001.