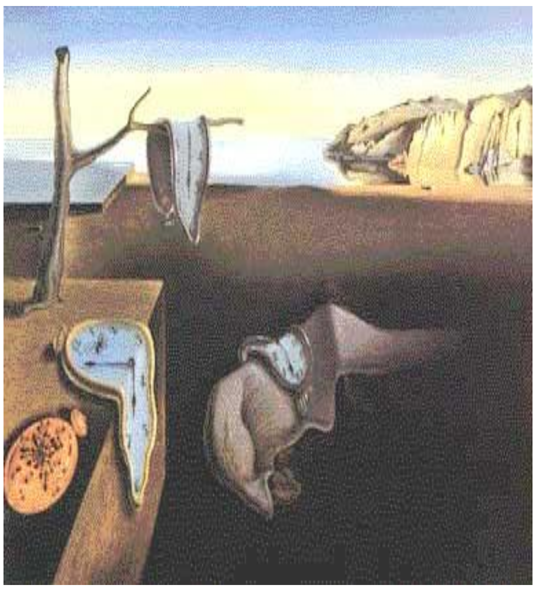
PHYS 3033 GENERAL RELATIVITY PART I
Chapter 2
Special relativity

A famous Zen story: Two Zen monks were arguing about a flag waving in the breeze, and whether it was the flag or the wind that was moving. The Sixth Patriarch of Zen, Hui Neng, overheard; "I suggested it was neither, that what moved was their own mind." [1]

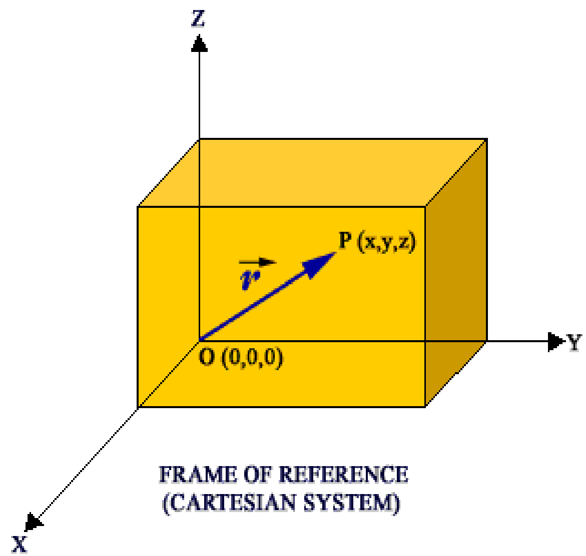
2.1 Measurement of length and time

All the measurement processes involve a process of comparison. We compare length to some standard length and intervals of time to some standard intervals of time. In fact each and every measurement involves two comparisons. If we wish to measure the length of the distance between two points we must align both of the points with the scale marks on a measuring road.

The description of the state of an object depends on the surroundings with respect to which it is specified. The surroundings themselves can be specified in terms of the rigid bodies around. Any of these rigid bodies, supposed to be extended in all space, together with a time recording device, a clock, is said to constitute a **reference system or a frame of reference.**



The melting clocks, painting by Salvador Dali [5] (1904-1989)



An artist and a physicist's view of the frame of reference

All measurements in space and time must be made with respect to such frames. In actual practice, for the purpose of space measurements, we attach a coordinate system to the rigid body. In what follows, the location of an object will always be specified by using rectangular Cartesian coordinate systems.

Next, we should consider who actually makes the measurement. According to the standard practice the person who makes the measurements is referred as "the observer".

Definition: an **observer** is an experimenter, who is equipped to make measurements of length and time on bodies which are moving, or at rest, relative to his own frame of reference.

An observer has no more information than can be obtained by her or his measurements.

Definition: the **rest frame** of a body or an observer is the frame of reference in which the body or observer is at rest. It is also referred to as the commoving frame[2].

Definition: **proper measurements** are measurements made on a body which are in the rest frame of that body. Thus, for example, we speak of **proper length** and **proper time**.

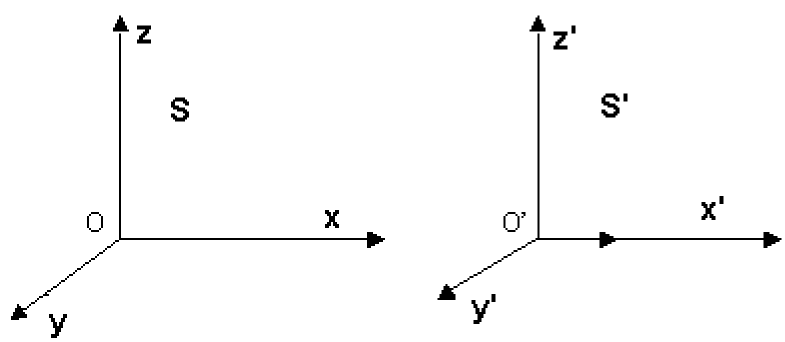
Ideal clocks and **ideal measuring roads** are postulated not to depend on the acceleration. A corollary of this idea is the principle that an accelerated observer makes the same local time and distance measurements as an inertial observer momentarily commoving with him.

Definition: A frame of reference in which a body satisfies Newton's first law is known as an **inertial frame**.

As the property of a particle which satisfies Newton's first law is knows as inertia, so it is natural to name a frame in which such a state can be observed as an inertial frame.

An important question is how inertial frames can be realized in practice. It is obvious that a rotating coordinate system would not be an inertial frame, as a freely moving particle would appear to travel in a curved path. The existence of a curved path implies the existence of a force. When we talk of a coordinate system fixed to the Earth, it is necessary to remember that the Earth rotates and inevitably the result is that we have a non-inertial frame. However, if we want to measure the period of a simple pendulum, then a frame fixed to the surface of the Earth is a perfectly adequate inertial frame, with the effects of rotation being almost undetectable.

Let us consider two frames S and S', as illustrated in the figure.



The frames have the following properties:

a) They are identical
b) S' moves relative to S with speed v
c) The motion of the origin of S' is along the x-axis of S
d) x and x' are co-axial
e) At $t = t' = 0$, the origins O and O' coincide.

This set-up is called the standard configuration. It can be extended to any number of frames S', S'', S''',…, with origins moving along the $x-$axis with speeds v, v', v'',…. In order to specify an event, we have to say where it happened and when it happened. Then we may consider how the event appears from two different inertial frames of reference S and S'. From now on we shall take the two frames to be in the standard

configuration. Obviously, there is only the one event, but its coordinates will be different in two different frames of reference, thus:
-in frame S, $E \equiv \text{the point}\,(x, y, z, t)$
-in frame S’, $E \equiv \text{the point}\,(x', y', z', t')$

It is easy to derive the relations between coordinates in the frames S and S’ in the framework of the Newtonian mechanics. Since the time is absolute, we have

$$t' = t\,.$$

For x and x', the relationship between the two coordinates has to take into account that the two origins coincided at $t = 0$, and thereafter separated at a speed v. The origin of S’ is located at $x = \mathrm{v}t$ at time t, and hence the x-direction coordinates are related by

$$\mathrm{v}t + x' = x\,,$$

or

$$x' = x - \mathrm{v}t\,.$$

Thus, collecting all the results for coordinate transformations between the two frames, we may summarize the relationship between the two observations of the same event as measured in S and S’ in the Newtonian mechanics as follows:

$$x' = x - \mathrm{v}t\,,$$

$$y' = y, z' = z, t' = t.$$

These are known as the standard Galilean transformations.
In a general vectorial form they can be written as

$$\vec{r}\,' = \vec{r} - \vec{\mathrm{v}}t\,.$$

Consider a particle moving at speed V along the $x-$axis in S. What is its speed V' in S’? Assume that a time t has elapsed from the moment of the beginning of motion. The particle has traveled a distance Vt in S and $V't'$ in S’, while S’ has traveled a distance $\mathrm{v}t$ in S. Then it follows that the displacement of the particle in one frame is related to its displacement in the other by

$$Vt = \mathrm{v}t + V't$$

or

$$V' = V - \mathrm{v}\,.$$

Thus in the classical Newtonian picture the velocity of any process will depend on the frame in which it is measured. The key element in obtaining the transformation law is that time is the same in both frames.

Formally the same result can be obtained by differentiating $\vec{r}\,'$ with respect to t:

$$\frac{d\vec{r}\,'}{dt} = \frac{d\vec{r}}{dt} - \vec{\mathrm{v}},$$

or

$$\vec{V}\,' = \vec{V} - \vec{\mathrm{v}},$$

where $\vec{V}\,' = \frac{d\vec{r}\,'}{dt}$ and $\vec{V} = \frac{d\vec{r}}{dt}$ denotes the velocities of the same particle as measured in the frames S and S'.

It is useful to use a fictitious four-dimensional space, on the axes of which are marked three space coordinates and the time. In this space events are represented by points, called world points. To each particle a certain line corresponds in the four-dimensional space, called a world line. The points of this line determine the coordinates of the particle at all moments of time. To a particle in uniform rectilinear motion there corresponds a straight world line.

2.2 Special relativity

By the end of the nineteenth century, there was a general view that physics as a subject was complete. The natural philosophy of the Universe was thought to be well understood. However, there were some cracks in this façade, which many people at the time thought could be patched up, but which were ultimately to turn into large fissures which would destroy the whole structure. The result was the intellectual revolution that gave us both quantum theory and special relativity.

There were two particular problems which led to special relativity [3].

Problem 1: The laws of propagation of electromagnetic waves (that is, Maxwell's equations) are not Galilean invariant.

Problem 2: The Michelson-Morley experiment implied that the velocity of light in vacuum, c, is independent of the frame of reference. This result violates the Galilean law of addition of velocities.

Faced, like the other physicists of his time, with the need to chose between two mutually exclusive choices, that is

- the correctness of Newton's law and Galilean invariance on the one hand; and
- the correctness of Maxwell's equations and Lorentz invariance on the other;

Einstein made his choice and enunciated it in the form of two postulates [3]:

Postulate 1: The laws of physics are the same in all inertial reference frames.

Postulate 2: The velocity of light in vacuum is the same in all inertial frames.
A corollary can be appended to these postulates as follows:

Corollary. No physical experiment can be used to tell whether an inertial frame is moving or at rest (with respect to any other frame).

2.3 Space-time intervals

We now express the principle of the invariance of the velocity of light in mathematical form. For this purpose we consider two standard reference systems S and S' moving relative to each other with constant velocity [4] .

Let the first event consist of sending out a signal, propagating with the speed of light, from a point having coordinates (x_1, y_1, z_1) in the S system, at the time t_1. We observe the propagation of the signal in S. Let the second event consists of the arrival of the signal at the point (x_2, y_2, z_2) at the moment of time t_2. Since the signal propagates with velocity c, the distance covered by it is $c(t_2 - t_1)$. On the other hand the same distance is $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Thus we can write the following relation between the coordinates of the two events in the S system:

$$(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2 - c^2(t_2 - t_1)^2 = 0.$$

The same two events, i.e. the propagation of the signal, can be also observed from the S' system. Let the coordinates of the first event in S' be (x_1', y_1', z_1', t_1') and of the second (x_2', y_2', z_2', t_2'). Since the velocity of light is the same in both S and S', we have

$$(x_2' - x_1')^2 + (y_2' - y_1')^2 + (z_2' - z_1')^2 - c^2(t_2' - t_1')^2 = 0.$$

If (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) are the coordinates of any two events in the frame S, then the quantity

$$s_{12} = \sqrt{c^2(t_2 - t_1)^2 - (x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2},$$

is called the interval between these two events.

Thus from the principle of the invariance of the speed of light it follows that if the interval between two events is zero in one coordinate system, then it is equal to zero in all other systems.

If two events are infinitely closed to each other, then the interval ds between them is

$$ds^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2.$$

As we have already shown, if $ds = 0$ in one inertial frame, then $ds' = 0$ in any other system. On the other hand ds and ds' are infinitesimals of the same order. From these two conditions it follows that ds^2 and ds'^2 must be proportional to each other:

$$ds^2 = a ds'^2 ,$$

where the coefficient a can depend only on the absolute value of the relative velocity of the two inertial systems. It cannot depend on the coordinates or the time, since then different points in space and different moments in time would not be equivalent, which would be in contradiction to the homogeneity of space and time. Similarly, it cannot depend on the direction of the relative velocity, since that would contradict the isotropy of space.

Let us consider three reference systems S, S' and S'' and let v_1 and v_2 be the velocities of systems S' and S'' relative to S. Then we have

$$ds^2 = a(\mathrm{v}_1) ds'^2 ,\ ds^2 = a(\mathrm{v}_2) ds''^2 .$$

Similarly we can write

$$ds'^2 = a(\mathrm{v}_{12}) ds''^2 ,$$

where v_{12} is the absolute value of the velocity of S'' with respect to S'. Comparing these relations with one another, we find that we must have

$$\frac{a(\mathrm{v}_2)}{a(\mathrm{v}_1)} = a(\mathrm{v}_{12}). \qquad (1)$$

But v_{12} depends not only on the absolute values of the vectors $\vec{\mathrm{v}}_1$ and $\vec{\mathrm{v}}_2$ but also on the angle between them. However, this angle does not appear on the left side of formula (1). It is therefore clear that this formula can be correct only if the function $a(\mathrm{v})$ reduces to a constant, which is equal to unity according to the same formula.

Thus

$$ds^2 = ds'^2 ,$$

and from the equality of the infinitesimal intervals there also follows the equality of finite intervals: $s = s'$.

Hence we arrived at a very important result:

The interval between two events is the same in all inertial systems of reference, that is, it is invariant under transformations from one inertial system to any other.

This invariance is the mathematical expression of the constancy of the velocity of light.

2.4 Timelike and spacelike intervals

Let (x_1, y_1, z_1, t_1) and (x_2, y_2, z_2, t_2) be the coordinates of two events in a certain reference system S. **Does there exist a coordinate system S', in which these two events occur at one and the same point in space?**

Let's introduce first the notation $t_{12} = t_2 - t_1$ and $l_{12} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$. Then the interval between events in the system S is

$$s_{12}^2 = c^2 t_{12}^2 - l_{12}^2.$$

In S' the interval is

$$s_{12}^{'2} = c^2 t_{12}^{'2} - l_{12}^{'2}.$$

Because of the invariance of intervals

$$c^2 t_{12}^2 - l_{12}^2 = c^2 t_{12}^{'2} - l_{12}^{'2}.$$

We want the two events to occur at the same point in the S' system, that is, we require $l_{12}^{'2} = 0$. Then

$$s_{12}^2 = c^2 t_{12}^2 - l_{12}^2. = c^2 t_{12}^{'2} > 0.$$

Consequently a system of reference with the required property exists if $s_{12}^2 > 0$, that is, if the interval between the two events is a real number. **Real intervals are said to be timelike.**

Thus, if the interval between two events is timelike, then there exists a system of reference in which the two events occur at one and the same place. The time which elapses between the two events in this system is

$$t_{12}^{'} = \frac{1}{c}\sqrt{c^2 t_{12}^2 - l_{12}^2} = \frac{s_{12}}{c}.$$

If two events occur in one and the same body, then the interval between them is always timelike, for the distance which the body moves between the two events cannot be greater than ct_{12}, since the velocity of the body cannot exceed c. So we always have

$$l_{12} < ct_{12}.$$

Let us now ask whether or not we can find a system of reference in which the two events occur at one and the same time. As before, we have for the S and S' systems

$$c^2 t_{12}^2 - l_{12}^2 = c^2 t_{12}^{'2} - l_{12}^{'2}.$$

We want now to have $t_{12}^{'} = 0$, so that

$$s_{12}^2 = -l_{12}^{'2} < 0.$$

Consequently the required system can be found only for the case when the interval s_{12} between the two events is an imaginary number. **Imaginary intervals are said to be space-like.**

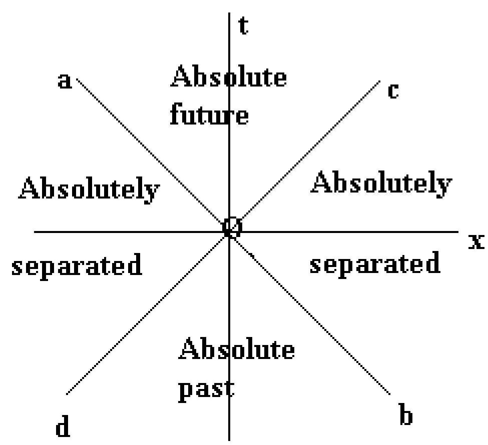
Thus if the interval between two events is space-like, there exists a reference system in which the two events occur simultaneously. The distance between the points where the events occur is in this system is

$$l_{12}^{'} = \sqrt{l_{12}^2 - c^2 t_{12}^2} = i s_{12}.$$

The division of intervals into space and time-like intervals is, because of their invariance, an absolute concept. This means that the time-like or space-like character of an interval is independent of the reference system.

Let us take some event O as origin of time and space coordinates. In other words, in the four-dimensional system of coordinates, the axes of which are marked (x, y, z, t) the world point of the event O is the origin of coordinates. Let us now consider what relation other events bear to the given event O.

For visualization, we shall consider only one space dimension and the time, marking them on two axes.



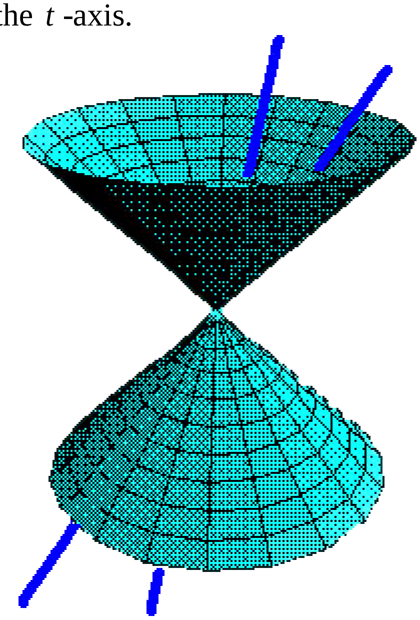
Uniform rectilinear motion of a particle, passing through $x = 0$ at $t = 0$ is represented by a straight line going through O and inclined to the t axis at an angle whose tangent is the velocity of the particle. Since the maximum possible velocity is c, there is a maximum angle which this line can subtend with the t axis. In the figure the two lines representing the propagation of two signals (with the velocity of light) in opposite directions passing through the event O (i.e. going through $x = 0$ at $t = 0$ are shown. All lines representing the motion of particles can lie only in the regions aOc and dOb. On the lines ab and cd $x = \pm ct$.

First consider events whose world points lie within the region aOc. For all points of this region $c^2t^2 - x^2 > 0$. In other words, the interval between any event in this region and the event O is timelike. In this region $t > 0$, i.e. all the events in this region occur after the event O. But two events which are separated by a time-like interval cannot occur simultaneously in any reference system. Consequently it is impossible to find a reference system in which any of the events in region aOc occurred before the event O, i.e. at the time $t < 0$. Thus all the events in the region aOc are future events relative to O in all reference systems. Therefore this region can be called the absolute future relative to O.

In exactly the same way all events in the region bOd are in the absolute past relative to O; i.e. events in this region occur before the event O in all systems of reference.

Next consider regions dOa and cOb. The interval between any event in this region and the event O is spacelike. These events occur at different points in space in every reference system. Therefore these regions can be said to be absolutely remote relative to O. However, the concepts "simultaneous", "earlier" and "later" are relative for these regions. For any event in these regions there exist systems of reference in which it occurs after the event O, systems in which it occurs earlier than O and finally one reference system in which it occurs simultaneously with O.

Note that if we consider all three space coordinates instead of just one, then instead of the two intersecting lines of the figure above we would have a cone $x^2 + y^2 + z^2 - c^2t^2 = 0$ in the four-dimensional coordinate system (x, y, z, t), the axis of the cone coinciding with the t-axis.



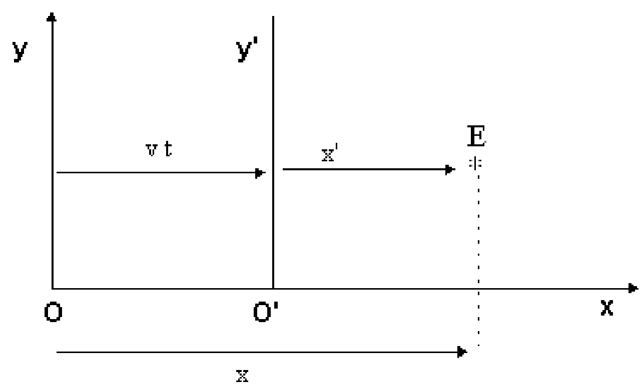
This cone is called the light cone. The regions of absolute future and absolute past are then represented by the two interior portions of the light cone.

Two events can be related causally to each other only if the interval between them is time-like; this follows immediately from the fact that no interaction can propagate with a velocity greater than the velocity of light.

2.5 The Lorentz transformation

Suppose that an event is described by the coordinates (x, y, z, t) in the system S. In another inertial frame S' the coordinate of the event are (x', y', z', t'). An important problem of the theoretical physics is to find the formula of transformation of the coordinates from the system S to the system S'.

The relativistic transformations can be obtained as a consequence that they leave the interval between events invariant. Let us consider that the S and S' frames are in the standard configuration. This means that the y and z coordinates do not change when passing from S to S'.



Therefore it follows that the transformation must leave unchanged the difference $c^2t^2 - x^2$, that is, we must have

$$c^2t^2 - x^2 = c^2t'^2 - x'^2.$$

The most general relation between the coordinates in the two frames of reference can be given as [5]

$$x = x'\cosh\varphi + ct'\sinh\varphi,\ ct = x'\sinh\varphi + ct'\cosh\varphi. \qquad (2)$$

The hyperbolic functions $\sinh x$, $\cosh x$, $\tanh x$ and $\coth x$ are defined as $\sinh x = \dfrac{e^x - e^{-x}}{2}$, $\cosh x = \dfrac{e^x + e^{-x}}{2}$, $\tanh x = \dfrac{\sinh x}{\cosh x}$ and $\coth x = \dfrac{\cosh x}{\sinh x}$, respectively. The functions $\sinh x$ and $\cosh x$ satisfy the basic relation $\cosh^2 x - \sinh^2 x = 1$.

Exercise. Show that the four-dimensional interval is invariant with respect to the transformations (2).

Let us consider the motion, in the S system, of the origin of the S' system. Then $x' = 0$ and formulas (2) take the form

$$x = ct'\sinh\varphi,\ ct = ct'\cosh\varphi.$$

Dividing one by other we obtain

$$\frac{x}{ct} = \tanh\varphi.$$

But $\frac{x}{t}$ is clearly the velocity v of the system S' with respect to S. Hence

$$\tanh\varphi = \frac{\mathrm{v}}{c}.$$

From this, by using the properties of the hyperbolic functions, we obtain

$$\sinh\varphi = \frac{\frac{\mathrm{v}}{c}}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}},\ \cosh\varphi = \frac{1}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}.$$

Substituting in (2) we find

$$x = \frac{x' + \mathrm{v}t'}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}},\ y = y',\ z = z',\ t = \frac{t' + \frac{\mathrm{v}}{c^2}x'}{\sqrt{1-\frac{\mathrm{v}^2}{c^2}}}. \qquad (3)$$

Equations (3) are called the Lorentz transformations and are of fundamental importance in physics.

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H. A. Lorentz (1853-1928)

Born in Arnhem, Netherlands, in 1853, Lorentz was a professor at the Leyden University. He refined Maxwell's electromagnetic theory. Before the existence of electrons was proved, Lorentz proposed that light waves were due to oscillations of an electric charge in the atom. For his mathematical theory of the electron he received the Nobel Prize in 1902. Lorentz is also famed for his work on the Fitzgerald- Lorentz contraction, which is a contraction in the length of an object at relativistic speeds. Lorentz transformations, which he introduced in 1904, form the basis of Einstein's special theory of relativity.

The inverse formulas, expressing (x', y', z', t') as a function of (x, y, z, t) are obtained by changing v to –v in Eqs. (3). This is because the system S moves with velocity –v relative to the S' system. Therefore we obtain

$$x' = \frac{x - \mathrm{v}t}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}, \; y' = y, \; z' = z, \; t' = \frac{t - \frac{\mathrm{v}x}{c^2}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}. \qquad (4)$$

For velocities v small compared with the velocity of light, we obtain

$$x' = x - \mathrm{v}t, \; y' = y, \; z' = z, \; t' = t. \qquad (5)$$

Eqs. (4) represent the Galilean transformations of coordinates.

The equations (4) representing the Lorentz transformations can be written in a matrix form as

$$X' = LX,$$

where

$$X' = \begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix}, \; X = \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix},$$

and

$$L = \begin{pmatrix} \gamma & -\gamma\frac{\mathrm{v}}{c} & 0 & 0 \\ -\gamma\frac{\mathrm{v}}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix},$$

where we denoted $\gamma = \dfrac{1}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}$.

The inverse transformation is given by

$$X = L^{-1}X',$$

where L^{-1} is the inverse of the matrix L, with the property $LL^{-1} = L^{-1}L = I$ (I is the unity matrix), and with elements given by

$$L^{-1} = \begin{pmatrix} \gamma & \gamma \frac{\mathrm{v}}{c} & 0 & 0 \\ \gamma \frac{\mathrm{v}}{c} & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Exercise. Find the form of the inverse of the matrix L describing the Lorentz transformations. What is the value of the determinant of L?

Finally, we mention another general property of the Lorentz transformations, which distinguishes them from Galilean transformations. The latter have the general property of commutativity, i.e. **the combined result of two successive Galilean transformations (with different velocities v_1 and v_2) does not depend on the order in which the transformations are performed**. On the other hand, **the result of two successive Lorentz transformations does depend, in general, on their order.**

2.6 Transformation of velocities

Suppose that the S' system moves relative to S with velocity v along the $x-$axis. Let $u_x = dx/dt$ be the component of the particle velocity in the S system and $u_x^{'} = dx'/dt'$ the velocity component of the same particle in the S' system. From the Lorentz transformations (3) we obtain

$$dx = \frac{dx' + \mathrm{v}dt'}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}, \; dy = dy', \; dz = dz', \; dt = \frac{dt' + \frac{\mathrm{v}}{c^2} dx'}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}.$$

Dividing the first equation by the fourth and introducing the velocities $\vec{u} = d\vec{r}/dt$ and $\vec{u}' = d\vec{r}'/dt'$ we find

$$u_x = \frac{u_x^{'} + \mathrm{v}}{1 + u_x^{'} \frac{\mathrm{v}}{c^2}}, \; u_y = \frac{u_y^{'} \sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}{1 + u_x^{'} \frac{\mathrm{v}}{c^2}}, \; u_z = \frac{u_z^{'} \sqrt{1 - \frac{\mathrm{v}^2}{c^2}}}{1 + u_x^{'} \frac{\mathrm{v}}{c^2}}.$$

These formulas determine the transformation of velocities. In the limiting case $c \to \infty$ they go over into the formulas of the classical mechanics. In the special case of the motion of a particle parallel to the x-axis $u_x = u$, $u_y = u_z = 0$. Then $u_y^{'} = u_z^{'} = 0$, $u_x^{'} = u'$, so that

$$u = \frac{u' + \mathrm{v}}{1 + u' \frac{\mathrm{v}}{c^2}}.$$

It is easy to convince oneself that the sum of two velocities each smaller or equal than the velocity of light is again not greater than the light velocity.

Exercise. Derive the law for time dilation in special relativity.

Exercise. Derive the law for length contraction in special relativity.

Notes

[1] Hui Neng (638-713), who became the great Sixth Patriarch of Ch'an (Japanese Zen) was born in Guangzhou, China. His most important work is The Sutra of Hui Neng. More information on Hui Neng can be found on the website www.theosociety.com.

[2] The definitions of the frame of reference, observer, standard frame and events follow the definitions in W. D. McComb, Dynamics and relativity, Oxford, Oxford University Press, 1999.

[3] For a discussion of the physical basis of special relativity see W. D. McComb, Dynamics and relativity, Oxford, Oxford University Press, 1999, R. Resnick and D. Halliday, Basic concepts in relativity and early quantum theory, New York, McMillan Publishing Company, 1992 and E. F. Taylor and J. A. Wheeler, Space-time physics, New York, W. H. Freeman and Company, 2001.

[4] The presentation of the properties of the interval is based on the similar discussion in L. D. Landau and E. M. Lifshitz, The classical theory of fields, Oxford, Butterworth-Heinemann, 1998.

[5] Detailed derivations of the Lorentz transformations are also presented in W. D. McComb, Dynamics and relativity, Oxford, Oxford University Press, 1999, R. Resnick and D. Halliday, Basic concepts in relativity and early quantum theory, New York, McMillan Publishing Company, 1992 and E. F. Taylor and J. A. Wheeler, Space-time physics, New York, W. H. Freeman and Company, 2001; at a qualitative level the consequences of the Lorentz transformations are discussed in E. F. Taylor and J. A. Wheeler, Space-time physics, New York, W. H. Freeman and Company, 2001 and W. Thomas Griffith, The physics of everyday phenomena, New York, McGraw Hill 2001.