SINGH & SINGH : MATHEMATICAL MODEL FOR DAMAGE ASSESSMENT OF COMPOSITE PANELS

$$\begin{bmatrix} Q_{11}(\theta) \\ Q_{12}(\theta) \\ Q_{22}(\theta) \\ Q_{16}(\theta) \\ Q_{26}(\theta) \\ Q_{66}(\theta) \end{bmatrix} = \begin{bmatrix} c^4 & 2c^2s^2 & s^4 \\ c^2s^2 & c^4 + s^4 & c^2s^2 \\ s^4 & 2c^2s^2 & c^4 \\ c^3s & -cs(c^2 - s^2) - cs^3 \\ cs^3 & cs(c^2 - s^2) - c^3s \\ c^2s^2 & -2c^2s^2 & c^2s^2 \end{bmatrix}$$

where $c = \cos \theta$ and $s = \sin \theta$.

Now, consider a laminate comprising n plies and let the angle between the fibre direction in the k-th ply and x-axis be θ_k . In terms of the average stresses (averaged through the thickness), the stress-strain law for the laminate becomes:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{bmatrix}$$
(22)

where

$$A_{ij} = (\sum_{k=1}^{n} Q_{ij}(\theta_k))/n \tag{23}$$

For orthotropic laminates, having structural axes as the axes of orthotropy, $A_{16} = A_{26} = 0$ (as in the case with a laminate consisting of 0° and matched pairs of $\pm \theta$ ° plies only). The moduli for orthotropic laminate can be obtained as⁸

$$E_x = A_{11} - A_{12}^2 / A_{22}, E_y = A_{22} - A_{12}^2 / A_{11},$$

$$v_{xy} = A_{12}/A_{22}, \ v_{yx} = A_{12}/A_{11}, \ G_{xy} = A_{66}$$
 (24)

Once the moduli are determined, the dynamic response of the laminate panel may be obtained as discussed in 2.1.

2.3.2 Calculation of Stresses and Failure Criterion for Individual Plies in a Laminate

Once the lay-up of plies in a laminate is known, the procedure described in 2.1 would give the deflection and hence the strains may be computed which are the same for all the plies. However, if the above procedure is used to obtain the stresses, then only average stresses are obtained. To determine the actual stresses in a particular ply, the strains obtained thus have to be substituted into the Eqn. (20). The actual stress distribution may be very different from the average one, as transverse direct stresses and shear stresses may develop even though no such stresses are applied to the laminate.

Thus, it is more reasonable to develop a failure criterion based on the actual stresses in the individual plies than the average stresses as it is quite possible that only a few plies at a particular orientation may be damaged under a given blast load while the majority of the plies may remain unaffected.

3. COMPARISON WITH CLASSICAL THEORY

Here, the methodology based on classical theory is described briefly. The details are given in⁹. The governing system of partial differential equations in this case is:

$$\frac{1}{E_{y}} \frac{\partial^{4} F}{\partial x^{4}} + \left(\frac{1}{E_{3}}\right) \frac{\partial^{4} F}{\partial x^{2} \partial y^{2}} + \frac{1}{E_{x}} \frac{\partial^{4} F}{\partial y^{4}}$$

$$= \left(\frac{\partial^{2} \omega}{\partial x \partial y}\right)^{2} - \frac{\partial^{2} \omega}{\partial x^{2}} \frac{\partial^{2} \omega}{\partial y^{2}} \tag{25}$$

$$D_{x} \frac{\partial^{4} \omega}{\partial x^{4}} + 2D_{3} \frac{\partial^{4} \omega}{\partial x^{2} \partial y^{2}} + D_{y} \frac{\partial^{4} \omega}{\partial y^{4}} + \rho h \frac{\partial^{2} \omega}{\partial t^{2}}$$

$$= p(x, y, t) + h \left[\frac{\partial^{2} F}{\partial y^{2}} \frac{\partial^{2} \omega}{\partial x^{2}} - 2 \frac{\partial^{2} F}{\partial x \partial y} \frac{\partial^{2} \omega}{\partial x \partial y} + \frac{\partial^{2} F}{\partial x^{2}} \frac{\partial^{2} \omega}{\partial y^{2}} \right]$$
(26)

where F is the Airy's stress function, and

$$1/E_3 = 1/G_{xy} - 2v_{xy}/E_x$$
, $D_3 = D_x v_{yx} + 2G_{xy}h^3/12$.

(a) SS panel: In this case, the solution is assumed in the form:

$$\omega(x, y, t) = hf(t)\cos\frac{\pi x}{a}\cos\frac{\pi y}{b}$$
 (27)