Module 11

Reasoning with uncertainty-Fuzzy Reasoning

Lesson 32

Fuzzy Reasoning - Continued

11.4 Fuzzy Inferencing

The process of fuzzy reasoning is incorporated into what is called a Fuzzy Inferencing System. It is comprised of three steps that process the system inputs to the appropriate system outputs. These steps are 1) Fuzzification, 2) Rule Evaluation, and 3) Defuzzification. The system is illustrated in the following figure.

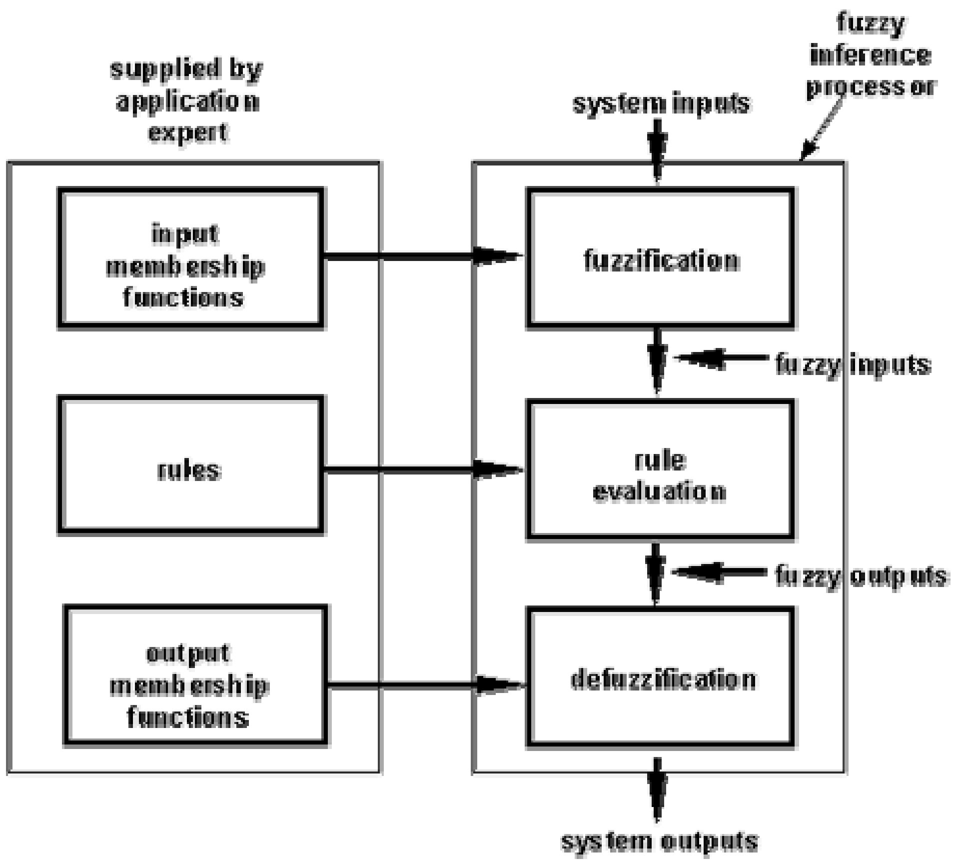


Figure 3: Fuzzy Inferencing Unit

Each step of fuzzy inferencing is described in the following sections.

11.4.1 Fuzzification

Fuzzification is the first step in the fuzzy inferencing process. This involves a domain transformation where crisp inputs are transformed into fuzzy inputs. Crisp inputs are exact inputs measured by sensors and passed into the control system for processing, such as temperature, pressure, rpm's, etc.. Each crisp input that is to be processed by the FIU has its own group of membership functions or sets to which they are transformed. This group of membership functions exists within a universe of discourse that holds all relevant values that the crisp input can possess. The following shows the structure of membership functions within a universe of discourse for a crisp input.

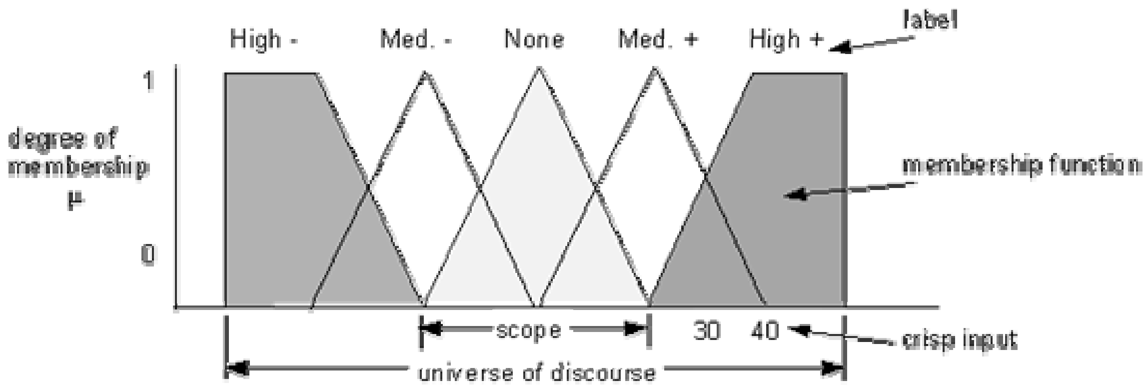


Figure 4: Membership Function Structure

where:

degree of membership: degree to which a crisp value is compatible to a membership function, value from 0 to 1, also known as truth value or fuzzy input.

membership function, MF: defines a fuzzy set by mapping crisp values from its domain to the sets associated degree of membership.

crisp inputs: distinct or exact inputs to a certain system variable, usually measured parameters external from the control system, e.g. 6 Volts.

label: descriptive name used to identify a membership function.

scope: or domain, the width of the membership function, the range of concepts, usually numbers, over which a membership function is mapped.

universe of discourse: range of all possible values, or concepts, applicable to a system variable.

When designing the number of membership functions for an input variable, labels must initially be determined for the membership functions. The number of labels correspond to the number of regions that the universe should be divided, such that each label describes a region of behavior. A scope must be assigned to each membership function that numerically identifies the range of input values that correspond to a label.

The shape of the membership function should be representative of the variable. However this shape is also restricted by the computing resources available. Complicated shapes require more complex descriptive equations or large lookup tables. The next figure shows examples of possible shapes for membership functions.

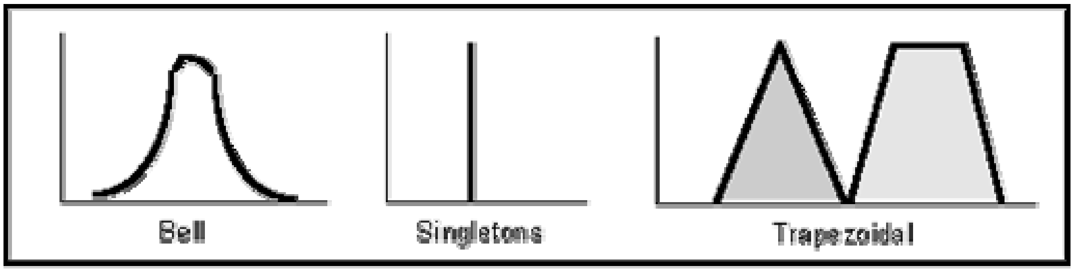


Figure 5: Membership Function Shapes

When considering the number of membership functions to exist within the universe of discourse, one must consider that:

i) too few membership functions for a given application will cause the response of the system to be too slow and fail to provide sufficient output control in time to recover from a small input change. This may also cause oscillation in the system.

ii) too many membership functions may cause rapid firing of different rule consequents for small changes in input, resulting in large output changes, which may cause instability in the system.

These membership functions should also be overlapped. No overlap reduces a system based on Boolean logic. Every input point on the universe of discourse should belong to the scope of at least one but no more than two membership functions. No two membership functions should have the same point of maximum truth, (1). When two membership functions overlap, the sum of truths or grades for any point within the overlap should be less than or equal to 1. Overlap should not cross the point of maximal truth of either membership function. Marsh has proposed two indices to describe the overlap of membership functions quantitatively. These are overlap ratio and overlap robustness. The next figure illustrates their meaning.

$$\text{Overlap Ratio} = \frac{\text{overlap scope}}{\text{adjacent MF scope}} \qquad (1)$$

$$\text{Overlap Robustness} = \frac{\text{area of summed overlap}}{\text{max. area of summed overlap}} = \frac{\int_L^U (\mu_1 + \mu_2)dx}{2(U-L)} \qquad (2)$$

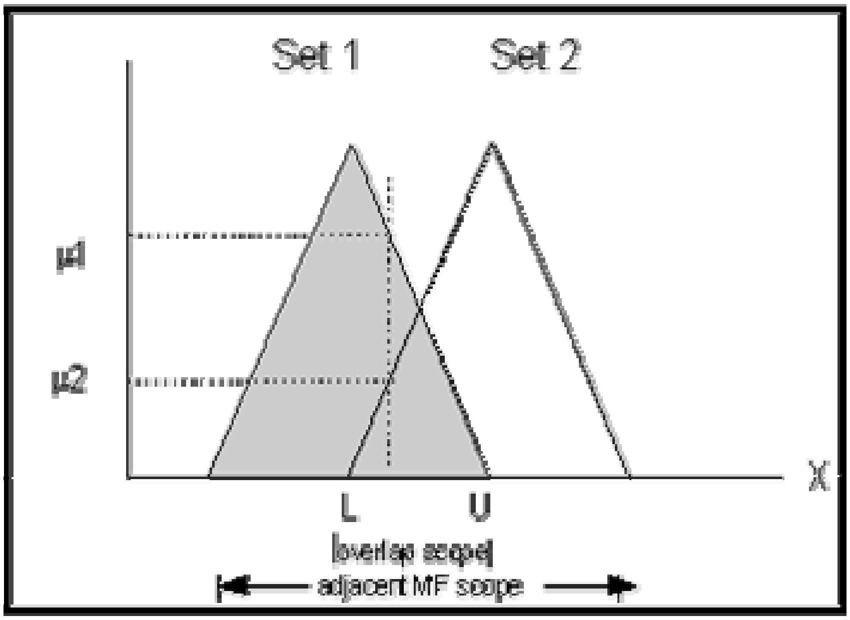


Figure 7: Overlap Indices

The fuzzification process maps each crisp input on the universe of discourse, and its intersection with each membership function is transposed onto the μ axis as illustrated in the previous figure. These μ values are the degrees of truth for each crisp input and are associated with each label as fuzzy inputs. These fuzzy inputs are then passed on to the next step, Rule Evaluation.

Fuzzy Rules

We briefly comment on so-called *fuzzy IF-THEN rules* introduced by Zadeh. They may be understood as partial imprecise knowledge on some crisp function and have (in the simplest case) the form IF x is A_i THEN y is B_i. They should **not** be immediately understood as implications; think of a *table* relating values of a (dependent) variable y to values of an (independent variable) x:

x	A_1	...	A_n
y	B_1	...	B_n

A_i, B_i may be crisp (concrete numbers) or fuzzy (small, medium, …) It may be understood in two, in general non-equivalent ways: (1) as a listing of n possibilities, called Mamdani's formula:

$$MAMD(x,y) \equiv \bigvee_{i=1}^{n} (A_i(x) \ \& \ B_i(y)).$$

(where x is A_1 and y is B_1 *or* x is A_2 and y is B_2 or …). (2) as a conjunction of implications:

$$RULES(x,y) \equiv \bigwedge_{i=1}^{n} (A_i(x) \rightarrow B_i(y)).$$

(if x is A_1 then y is B_1 *and* …).

Both *MAMD* and *RULES* define a binary fuzzy relation (given the interpretation of A_i's, B_i's and truth functions of connectives). Now given a *fuzzy input* $A^*(x)$ one can consider the image B^* of $A^*(x)$ under this relation, i.e.,

$$B^*(y) \equiv \exists x(A(x) \ \& \ R(x,y)),$$

where $R(x,y)$ is $MAMD(x,y)$ (most frequent case) or $RULES(x,y)$. Thus one gets an operator assigning to each fuzzy input set A^* a corresponding fuzzy output B^*. Usually this is combined with some *fuzzifications* converting a crisp input x_0 to some fuzzy $A^*(x)$ (saying something as "x is similar to x_0") and a *defuzzification* converting the fuzzy image B^* to a crisp output y_0. Thus one gets a crisp function; its relation to the set of rules may be analyzed.

11.4.2 Rule Evaluation

Rule evaluation consists of a series of IF-Zadeh Operator-THEN rules. A decision structure to determine the rules require familiarity with the system and its desired operation. This knowledge often requires the assistance of interviewing operators and experts. For this thesis this involved getting information on tremor from medical practitioners in the field of rehabilitation medicine.

There is a strict syntax to these rules. This syntax is structured as:

IF antecedent 1 ZADEH OPERATOR antecedent 2 .... THEN consequent 1 ZADEH OPERATOR consequent 2..............

The antecedent consists of: input variable IS label, and is equal to its associated fuzzy input or truth value $\mu(x)$.

The consequent consists of: output variable IS label, its value depends on the Zadeh Operator which determines the type of inferencing used. There are three Zadeh Operators, AND, OR, and NOT. The label of the consequent is associated with its output membership function. The Zadeh Operator is limited to operating on two membership functions, as discussed in the fuzzification process. Zadeh Operators are similar to Boolean Operators such that:

AND represents the intersection or *minimum* between the two sets, expressed as:

$$\mu_{A \cap B} = \min[\mu_A(x), \mu_B(x)]$$

OR represents the union or *maximum* between the two sets, expressed as:

$$\mu_{A \cup B} = \max[\mu_A(x), \mu_B(x)]$$

NOT represents the opposite of the set, expressed as:

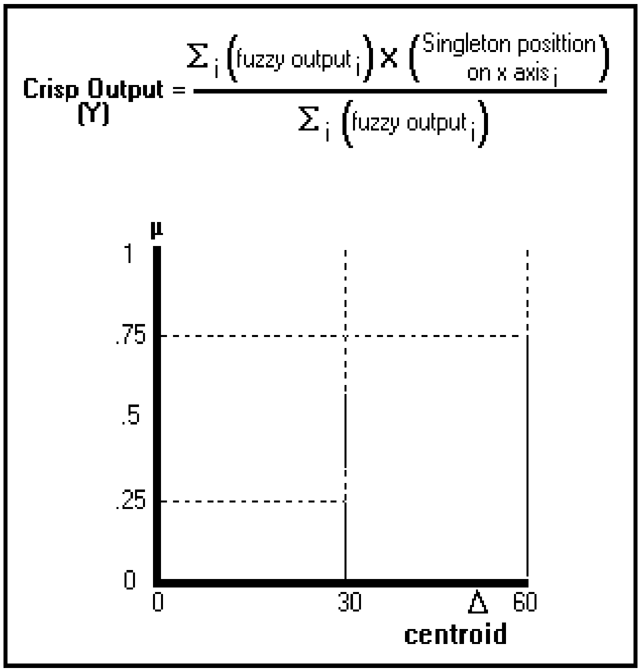
$$\overline{\mu_A} = [1 - \mu_A(x)]$$

The process for determining the result or rule strength of the rule may be done by taking the minimum fuzzy input of antecedent 1 AND antecedent 2, min. inferencing. This minimum result is equal to the consequent rule strength. If there are any consequents that are the same then the maximum rule strength between similar consequents is taken, referred to as maximum or max. inferencing, hence min./max. inferencing. This infers that the rule that is most true is taken. These rule strength values are referred to as fuzzy outputs.

11.4.3 Defuzzification

Defuzzification involves the process of transposing the fuzzy outputs to crisp outputs. There are a variety of methods to achieve this, however this discussion is limited to the process used in this thesis design.

A method of averaging is utilized here, and is known as the Center of Gravity method or COG, it is a method of calculating centroids of sets. The output membership functions to which the fuzzy outputs are transposed are restricted to being singletons. This is so to limit the degree of calculation intensity in the microcontroller. The fuzzy outputs are transposed to their membership functions similarly as in fuzzification. With COG the singleton values of outputs are calculated using a weighted average, illustrated in the next figure. The crisp output is the result and is passed out of the fuzzy inferencing system for processing elsewhere.



11.5 APPLICATIONS

Areas in which fuzzy logic has been successfully applied are often quite concrete. The first major commercial application was in the area of cement kiln control, an operation which requires that an operator monitor four internal states of the kiln, control four sets of operations, and dynamically manage 40 or 50 "rules of thumb" about their interrelationships, all with the goal of controlling a highly complex set of chemical interactions. One such rule is "If the oxygen percentage is rather high and the free-lime and kiln- drive torque rate is normal, decrease the flow of gas and slightly reduce the fuel rate".

Other applications which have benefited through the use of fuzzy systems theory have been information retrieval systems, a navigation system for automatic cars, a predicative fuzzy-logic controller for automatic operation of trains, laboratory water level controllers, controllers for robot arc-welders, feature-definition controllers for robot vision, graphics controllers for automated police sketchers, and more.

Expert systems have been the most obvious recipients of the benefits of fuzzy logic, since their domain is often inherently fuzzy. Examples of expert systems with fuzzy logic central to their control are decision-support systems, financial planners, diagnostic systems for determining soybean pathology, and a meteorological expert system in China for determining areas in which to establish rubber tree orchards.

Questions

1. In a class of 10 students (the universal set), 3 students speaks German to some degree, namely Alice to degree 0.7, Bob to degree 1.0, Cathrine to degree 0.4. What is the size of the subset *A* of German speaking students in the class?

2. In the above class, argue that the fuzzy subset *B* of students speaking a *very good* German is a fuzzy subset of *A*.

3. Let A and B be fuzzy subsets of a universal set X. Show that

$$|A \cup B| = |A| + |B| - |A \cap B|$$

4. For arbitrary fuzzy subsets *A* and *B*, show that

$$\overline{A \cup B} = \overline{A} \cap \overline{B}$$

5. Let X = {0, 1, 2, …., 6}, and let two fuzzy subsets, A and B, of X be defined by:

x	0	1	2	3	4	5	6
$\mu_A(x)$	1	0,7	0	1	0,5	0	0,4
$\mu_B(x)$	0,9	0,7	1	0,2	0,8	0,3	0

Notice that the description in a table as above is just a convenient notation for a fuzzy set description on the form: $A = \sum_i \left(\mu_A(x_i) / x_i \right)$. In its notation, the fuzzy set A in the example in this exercise is described by: $A = 1/0 + 0.7/1 + 1/3 + 0.5/4 + 0.4/6$.

Find:

$A \cap B, A \cup B, \overline{A}$ and $\overline{B}$

Solutions

1. |A| = 0.7 + 1.0 + 0.4 = 2.1

2. The addition of "very" strengthens the requirement, which consequently will be less satisfied. Thus for all $x \in X$, $B(x) \leq A(x)$, which is precisely what characterized the fuzzy subset relation $B \subseteq A$

3. Answer

$$|A \cup B| = |A| + |B| - |A \cap B|$$

$$\Leftrightarrow \quad |A \cup B| + |A \cap B| = |A| + |B| \qquad \text{(we added } |A \cap B| \text{ on both sides of =)}$$

$$\Leftrightarrow \quad \sum_{x \in X} \max(A(x), B(x)) + \sum_{x \in X} \min(A(x), B(x)) = \sum_{x \in X} A(x) + \sum_{x \in X} B(x) \qquad \text{(per definition of } |\cdot|\text{)}$$

For an arbitrary element x we have:

if $A(x) < B(x)$, then $\max(A(x), B(x)) + \min(A(x), B(x)) = B(x) + A(x)$

if $A(x) \geq B(x)$, then $\max(A(x), B(x)) + \min(A(x), B(x)) = A(x) + B(x)$

and therefore, for all x,

$$\max(A(x), B(x)) + \min(A(x), B(x)) = A(x) + B(x)$$

and

$$\sum_{x \in X} \left(\max(A(x), B(x)) + \min(A(x), B(x))\right) = \sum_{x \in X} \left(A(x) + B(x)\right)$$

$$\Leftrightarrow \quad \sum_{x \in X} \max(A(x), B(x)) + \sum_{x \in X} \min(A(x), B(x)) = \sum_{x \in X} A(x) + \sum_{x \in X} B(x).$$

4. Answer

$$\overline{A \cup B} : \mu_{\overline{A \cup B}}(x) = 1 - \max(\mu_A(x), \mu_B(x)) \quad \text{(for alle } x\text{)}$$

$$\overline{A} \cap \overline{B} : \mu_{\overline{A} \cap \overline{B}}(x) = \min(\mu_{\overline{A}}(x), \mu_{\overline{B}}(x)) = \min(1 - \mu_A(x), 1 - \mu_B(x))$$

$$= \begin{cases} 1 - \mu_A(x) & \text{if } \mu_A(x) \geq \mu_B(x) \\ 1 - \mu_B(x) & \text{if } \mu_B(x) \geq \mu_A(x) \end{cases} = 1 - \max(\mu_A(x), \mu_B(x))$$

5. Answer

x	0	1	2	3	4	5	6
$\mu_{A \cap B}(x)$	0,9	0,7	0	0,2	0,5	0	0
$\mu_{A \cup B}(x)$	1	0,7	1	1	0,8	0,3	0,4
$\mu_{\overline{A}}(x)$	0	0,3	1	0	0,5	1	0,6
$\mu_{\overline{B}}(x)$	0,1	0,3	0	0,8	0,2	0,7	1