Module 11

Reasoning with uncertainty-Fuzzy Reasoning

Lesson
31

Fuzzy Set Representation

11.3 Fuzzy Sets: BASIC CONCEPTS

The notion central to fuzzy systems is that truth values (in fuzzy logic) or membership values (in fuzzy sets) are indicated by a value on the range [0.0, 1.0], with 0.0 representing absolute Falseness and 1.0 representing absolute Truth. For example, let us take the statement:

"Jane is old."

If Jane's age was 75, we might assign the statement the truth value of 0.80. The statement could be translated into set terminology as follows:

"Jane is a member of the set of old people."

This statement would be rendered symbolically with fuzzy sets as:

mOLD(Jane) = 0.80

where m is the membership function, operating in this case on the fuzzy set of old people, which returns a value between 0.0 and 1.0.

At this juncture it is important to point out the distinction between fuzzy systems and probability. Both operate over the same numeric range, and at first glance both have similar values: 0.0 representing False (or non- membership), and 1.0 representing True (or membership). However, there is a distinction to be made between the two statements: The probabilistic approach yields the natural-language statement, "There is an 80% chance that Jane is old," while the fuzzy terminology corresponds to "Jane's degree of membership within the set of old people is 0.80." The semantic difference is significant: the first view supposes that Jane is or is not old (still caught in the Law of the Excluded Middle); it is just that we only have an 80% chance of knowing which set she is in. By contrast, fuzzy terminology supposes that Jane is "more or less" old, or some other term corresponding to the value of 0.80. Further distinctions arising out of the operations will be noted below.

The next step in establishing a complete system of fuzzy logic is to define the operations of EMPTY, EQUAL, COMPLEMENT (NOT), CONTAINMENT, UNION (OR), and INTERSECTION (AND). Before we can do this rigorously, we must state some formal definitions:

Definition 1: Let X be some set of objects, with elements noted as x. Thus, X = {x}.

Definition 2: A fuzzy set A in X is characterized by a membership function mA(x) which maps each point in X onto the real interval [0.0, 1.0]. As mA(x) approaches 1.0, the "grade of membership" of x in A increases.

Definition 3: A is EMPTY iff for all x, mA(x) = 0.0.

Definition 4: A = B iff for all x: mA(x) = mB(x) [or, mA = mB].

Definition 5: mA' = 1 - mA.

Definition 6: A is CONTAINED in B iff mA <= mB.

Definition 7: C = A UNION B, where: mC(x) = MAX(mA(x), mB(x)).

Definition 8: C = A INTERSECTION B where: mC(x) = MIN(mA(x), mB(x)).

It is important to note the last two operations, UNION (OR) and INTERSECTION (AND), which represent the clearest point of departure from a probabilistic theory for sets to fuzzy sets. Operationally, the differences are as follows:

For independent events, the probabilistic operation for AND is multiplication, which (it can be argued) is counterintuitive for fuzzy systems. For example, let us presume that x = Bob, S is the fuzzy set of smart people, and T is the fuzzy set of tall people. Then, if mS(x) = 0.90 and uT(x) = 0.90, the probabilistic result would be:

mS(x) * mT(x) = 0.81

whereas the fuzzy result would be:

MIN(uS(x), uT(x)) = 0.90

The probabilistic calculation yields a result that is lower than either of the two initial values, which when viewed as "the chance of knowing" makes good sense.

However, in fuzzy terms the two membership functions would read something like "Bob is very smart" and "Bob is very tall." If we presume for the sake of argument that "very" is a stronger term than "quite," and that we would correlate "quite" with the value 0.81, then the semantic difference becomes obvious. The probabilistic calculation would yield the statement

If Bob is very smart, and Bob is very tall, then Bob is a quite tall, smart person.

The fuzzy calculation, however, would yield

If Bob is very smart, and Bob is very tall, then Bob is a very tall, smart person.

Another problem arises as we incorporate more factors into our equations (such as the fuzzy set of heavy people, etc.). We find that the ultimate result of a series of AND's approaches 0.0, even if all factors are initially high. Fuzzy theorists argue that this is wrong: that five factors of the value 0.90 (let us say, "very") AND'ed together, should yield a value of 0.90 (again, "very"), not 0.59 (perhaps equivalent to "somewhat").

Similarly, the probabilistic version of A OR B is (A+B - A*B), which approaches 1.0 as additional factors are considered. Fuzzy theorists argue that a sting of low membership grades should not produce a high membership grade instead, the limit of the resulting membership grade should be the strongest membership value in the collection.

The skeptical observer will note that the assignment of values to linguistic meanings (such as 0.90 to "very") and vice versa, is a most imprecise operation. Fuzzy systems, it should be noted, lay no claim to establishing a formal procedure for assignments at this level; in fact, the only argument for a particular assignment is its intuitive strength. What fuzzy logic does propose is to establish a formal method of operating on these values, once the primitives have been established.

11.3.1 HEDGES

Another important feature of fuzzy systems is the ability to define "hedges," or modifier of fuzzy values. These operations are provided in an effort to maintain close ties to natural language, and to allow for the generation of fuzzy statements through mathematical calculations. As such, the initial definition of hedges and operations upon them will be quite a subjective process and may vary from one project to another. Nonetheless, the system ultimately derived operates with the same formality as classic logic.

The simplest example is in which one transforms the statement "Jane is old" to "Jane is very old." The hedge "very" is usually defined as follows:

m"very"A(x) = mA(x)^2

Thus, if mOLD(Jane) = 0.8, then mVERYOLD(Jane) = 0.64.

Other common hedges are "more or less" [typically SQRT(mA(x))], "somewhat," "rather," "sort of," and so on. Again, their definition is entirely subjective, but their operation is consistent: they serve to transform membership/truth values in a systematic manner according to standard mathematical functions.

A more involved approach to hedges is best shown through the work of Wenstop in his attempt to model organizational behavior. For his study, he constructed arrays of values for various terms, either as vectors or matrices. Each term and hedge was represented as a 7-element vector or 7x7 matrix. He ten intuitively assigned each element of every vector and matrix a value between 0.0 and 1.0, inclusive, in what he hoped was intuitively a consistent manner. For example, the term "high" was assigned the vector

0.0 0.0 0.1 0.3 0.7 1.0 1.0

and "low" was set equal to the reverse of "high," or

1.0 1.0 0.7 0.3 0.1 0.0 0.0

Wenstop was then able to combine groupings of fuzzy statements to create new fuzzy statements, using the APL function of Max-Min matrix multiplication.

These values were then translated back into natural language statements, so as to allow fuzzy statements as both input to and output from his simulator. For example, when the program was asked to generate a label "lower than sortof low," it returned "very low;" "(slightly higher) than low" yielded "rather low," etc.

The point of this example is to note that algorithmic procedures can be devised which translate "fuzzy" terminology into numeric values, perform reliable operations upon those values, and then return natural language statements in a reliable manner.